

# STAT 511: Summer 2018 – Quiz 2

Name: \_\_\_\_\_

Class Time: \_\_\_\_\_

Let  $X \sim N(1,1)$  and  $Y = |X - c|$  for some  $c$ .

1. (1 point) What is the support of  $Y$ ?

$(0, \infty)$

2. (2 points) Find  $P(Y \leq 2)$  when  $c = 1.5$ , i.e. the probability that  $X$  is within distance 2 from 1.5.

$$P(Y \leq 2) = P(|X - c| \leq 2) = P(c - 2 \leq X \leq c + 2) = P(c - 3 \leq X - 1 \leq c + 1) = \Phi(c + 1) - \Phi(c - 3)$$

(As  $X - 1 \sim N(0, 1)$ )

If  $c = 1.5$ ,

$$P(Y \leq 2) = \Phi(2.5) - \Phi(-1.5) = \Phi(2.5) + \Phi(1.5) - 1 = 0.9938 + 0.9332 - 1 = 0.927$$

3. (2 points) Using ideas from question 2, find a general form for the CDF of  $Y$ , i.e.  $P(Y \leq y)$  for a general  $y$  (involving  $c$  and the CDF  $\Phi(z)$  of  $Z$ ).

$$F_Y(y) = \begin{cases} 0, & \text{if } y \leq 0 \\ P(|X - c| \leq y) = P(c - y \leq X \leq c + y) = P(c - y - 1 \leq X - 1 \leq c + y - 1) = \Phi(c + y - 1) - \Phi(c - y - 1), & \text{if } y > 0 \end{cases}$$

4. (3 points) Find the median of  $Y$  when  $c = 1$ . Hint: use symmetry of  $\Phi(z)$ .

Solve:  $F_Y(y) = 0.5 \ (y > 0)$

$$\Rightarrow \Phi(y) - \Phi(-y) = 0.5 \quad (\text{Putting } c = 1 \text{ in Q3})$$

$$\Rightarrow 2\Phi(y) - 1 = 0.5$$

$$\Rightarrow \Phi(y) = 0.75 \Rightarrow y = z_{.75} = 0.67, \text{ i.e. } \tilde{\mu}_Y = 0.67$$

(From software,  $\tilde{\mu}_Y = 0.6745$ )

5. (2 points) Find  $E(Y^2)$  when  $c = 1$ . You don't need to do any integration.

$$E(Y^2) = E(X - 1)^2 \text{ (as } c = 1) = V(X) = 1$$

6. (Bonus: 2 points) Find the PDF of  $Y$  (involving  $c$  and the PDF  $\phi(z)$  of  $Z$ ). [Hint:  $\Phi'(z) = \phi(z)$ ]

$$f_Y(y) = F_Y'(y) = \begin{cases} 0, & \text{if } y \leq 0 \\ \Phi'(c + y - 1) \frac{d}{dy}(c + y - 1) - \Phi'(c - y - 1) \frac{d}{dy}(c - y - 1) \\ = \phi(c + y - 1) + \phi(c - y - 1), & \text{if } y > 0 \end{cases}$$

This is a 'Folded Normal Distribution' and reduces to a 'Half Normal Distribution' when  $c = 1$ .